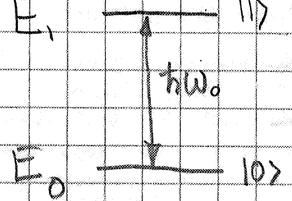
Qubit



ex: Spin in a magnetic field

$$\mathcal{H} = -\vec{\mu}\,\vec{B}$$

$$\mathcal{H}_0 = \frac{\hbar\omega_0}{2}\,\sigma_z$$

Pauli Matices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

state of the qubit

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle \qquad a_i \in \mathbb{C}$$

$$1 = \langle\psi|\psi\rangle = |a_0|^2 + |a_1|^2$$

global phase

$$|\psi\rangle = \cos(\theta/2)\,|0\rangle + e^{i\varphi}\sin(\theta/2)\,|1\rangle$$

$x = \sin(\theta)\cos(\varphi)$

$y = \sin(\theta)\sin(\varphi)$

$z = \cos(\theta)$

Example: $\theta = \frac{\pi}{2}$ $\varphi = \frac{\pi}{2}$

$\Rightarrow (0, 1, 0)$

$$|y\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + i\,|1\rangle\right)$$

Schrödinger Equation

$$i\hbar \frac{d}{dt}|\psi\rangle = H|\psi\rangle$$

time evolution of state
of a closed quantum system

$a_0(t) = e^{-i\omega_0/2\,t}\, a_0(0)$

$a_1(t) = e^{+i\omega_0/2\,t}\, a_1(0)$

$\varphi(t) = \omega_0 t + \varphi(0)$

$\theta(t) = \theta(t)$

Larmor precession

External oscillating field

$$\mathcal{H} = \mathcal{H}_0 + \frac{W}{2}\left(\sigma_x \cos(\omega t) + \sigma_y \sin(\omega t)\right)$$

example $\vec{B} = \vec{B}_0 + \vec{B}_1 \cos(\omega t)$

$$\mathcal{H} = \frac{\hbar}{2}\begin{pmatrix} \omega_0 & \omega_1 e^{-i\omega t} \\ \omega_1 e^{i\omega t} & -\omega_0 \end{pmatrix}$$

$$\hbar\omega_1 := W$$

$$i\frac{d}{dt}a_1(t) = \frac{\omega_0}{2}a_1(t) + \frac{\omega_1}{2}e^{-i\omega t}a_0(t)$$

$$i\frac{d}{dt}a_0(t) = \frac{\omega_1}{2}e^{i\omega t}a_1(t) - \frac{\omega_0}{2}a_0(t)$$

changing to a rotating frame

$$b_1(t) = a_1(t)\,e^{i\omega t/2}$$

$$b_0(t) = a_0(t)\,e^{-i\omega t/2}$$

$$i \frac{d}{dt} b_1(t) = -\frac{\Delta\omega}{2} b_1(t) + \frac{\omega_1}{2} b_0(t)$$

$$i \frac{d}{dt} b_0(t) = \frac{\omega_1}{2} b_1(t) + \frac{\Delta\omega}{2} b_0(t)$$

$$\Delta\omega = \omega_0 - \omega$$

$$i\hbar \frac{d}{dt} |\hat{\psi}(t)\rangle = \tilde{H} \, |\hat{\psi}(t)\rangle$$

$$\tilde{H} = \begin{pmatrix} -\Delta\omega & \omega_1 \\ \omega_1 & \Delta\omega \end{pmatrix}$$

$$|\hat{\psi}(t)\rangle = R(t) \, |\psi(t)\rangle$$

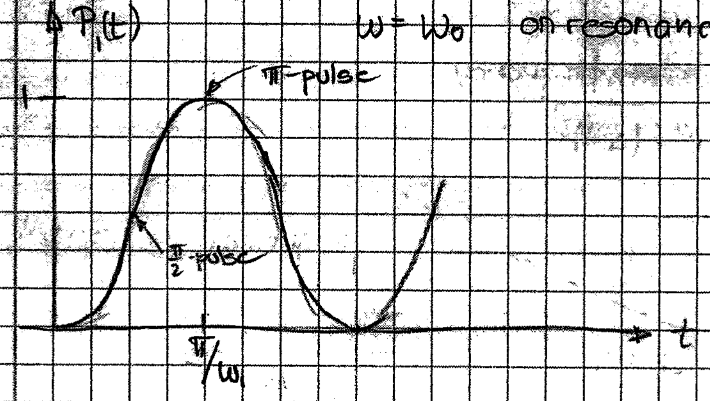
$$R(t) = e^{i\omega t \sigma_z / \hbar}$$

L2.3

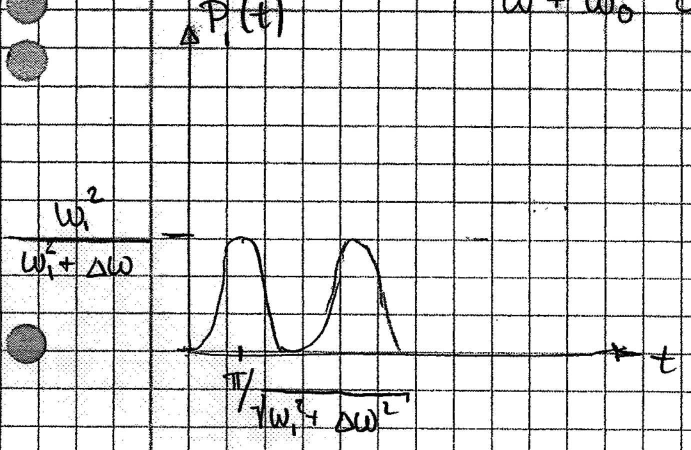
$$P_1(t) = |\langle 1 | \psi(t) \rangle|^2 = |\langle 1 | \tilde{\psi}(t) \rangle|^2$$
$$= |b_1(t)|^2$$

$$P_1(t) = \frac{\omega_1^2}{\omega_1^2 + (\Delta\omega)^2} \sin^2\left(\sqrt{\omega_1^2 + (\Delta\omega)^2}\,\frac{t}{2}\right)$$

$\omega = \omega_0$ on resonance

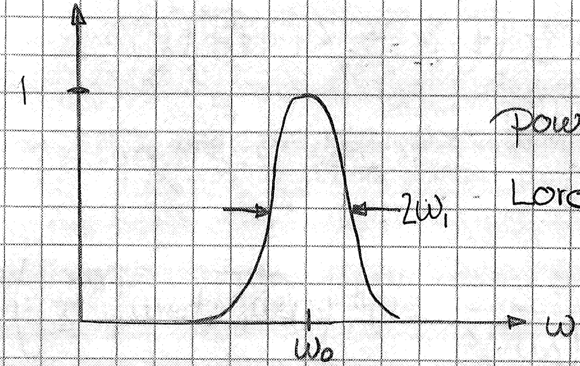


$\omega \neq \omega_0$ off resonance

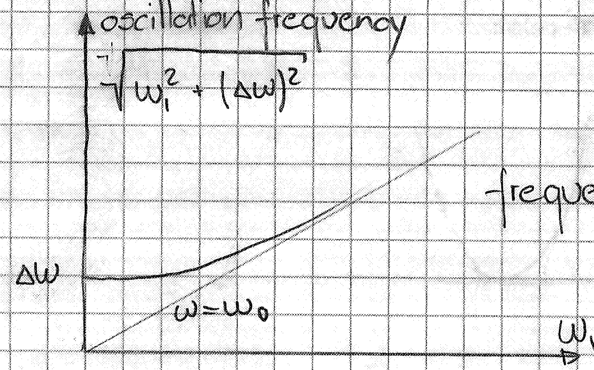


Power broadening

Lorentz curve



frequency ∝ amplitude



L2.4

Open quantum system

density matrix

ensemble of pure states $|\psi_i\rangle$ with probability p_i

$$\rho = \sum p_i \, |\psi_i\rangle\langle\psi_i|$$

general properties

$$\boxed{tr(\rho) = 1}$$

$$tr(\rho) = \sum p_i \, tr\left(|\psi_i\rangle\langle\psi_i|\right) = \sum p_i = 1$$

$\boxed{\rho \text{ is a positive, hermitian}}$

$\langle\varphi|\rho|\varphi\rangle \geq 0 \qquad \rho = \rho^\dagger$

pure state $\rho^2 = \rho$

$$\mathrm{tr}(\rho^2) \leq 1 \quad \underline{\text{general}}$$

$$= 1 \iff \text{pure state}$$

$$< 1 \iff \text{mixed state}$$

expectation value of an operator

$$\langle A \rangle = \mathrm{tr}(\rho A)$$

<u>spin 1/2</u>

pure state

$$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix} \cos^2(\theta/2) & \sin(\theta/2)\cos(\theta/2)e^{-2i\varphi} \\ \sin(\theta/2)\cos(\theta/2)e^{2i\varphi} & \sin^2(\theta/2) \end{pmatrix}$$

examples $\quad |1\rangle \rightarrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$$|x\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \longrightarrow \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\mathrm{tr}(\rho_{|x\rangle}^2) = 1$$

mixed state

$$\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

$$\mathrm{tr}\left(\rho_m^2\right) = \frac{1}{2}$$

general $\quad \rho \in \mathbb{C}(2\times 2)$

$$\rho = \alpha_0 \mathbb{1} + \vec{\alpha}\,\vec{\sigma}$$
$$= \alpha_0 \mathbb{1} + \alpha_x \sigma_x + \alpha_y \sigma_y + \alpha_z \sigma_z$$

ρ density matrix

$$\rho = \frac{1}{2}\left(\mathbb{1} + \vec{s}\,\vec{\sigma}\right)$$

$$s_x = \langle \sigma_x \rangle = \mathrm{tr}\left(\rho\,\sigma_x\right)$$
$$s_y = \langle \sigma_y \rangle = \mathrm{tr}\left(\rho\,\sigma_y\right)$$
$$s_z = \langle \sigma_z \rangle = \mathrm{tr}\left(\rho\,\sigma_z\right)$$

$$\mathrm{tr}(\rho^2) \leq 1 \implies s_x^2 + s_y^2 + s_z^2 \leq 1$$

$= 1$ pure state

dynamics

Von Neumann equation

$$i\hbar \frac{d\rho}{dt} = [H,\rho] \qquad [a,b] = ab - ba$$

unitary evolution

Lindblad master equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \sum_j \left(2L_j \rho L_j^\dagger - \{L_j^\dagger L_j, \rho\}\right)$$

markovian

memoryless

Note: $tr(\rho(t)) = 1 \qquad \rho(t) = \rho^\dagger(t)$